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## NUMBER THEORY.

**217. (May, 1914.) Proposed by E. T. BELL, University of Washington.**

(i) If $r$ is a prime greater than 2, and $p \equiv 2^a r + 1$ is prime, the only solution, when $n$ is greater than 2, of $x^n - y^n = p$, is $n = 3$, $x = 2$, $y = 1$.

(ii) The only primes that are simultaneously of the forms $4k + 1$ and $3^m - 2^m$ are 1 and 5.

(iii) Generalize (ii).

### SOLUTION BY FRANK IRWIN, University of California.

(i) Since $x^n - y^n$ is divisible by $x - y$, we must, if it is to be a prime, have $x - y = 1$, or $x = y + 1$.

Again, $n$ must be a prime; for $x^{st} - y^{st}$ is divisible by $x^s - y^s$, and cannot, therefore, be a prime.

Now $2^a r = p - 1 = x^n - y^n - 1 = (y + 1)^n - y^n - 1 = ny^{n-1} + \binom{n}{2} y^{n-2} + \cdots + ny$; so that $2^a r$ is divisible by $ny$, since $n$, being a prime, divides each of the binomial coefficients. Consequently we must have $n = r$ ($n$ is greater than 2 by hypothesis) and $y$ a power of 2.

But by writing $2^a r$ as $x^n - (x - 1)^n - 1$, we may show, just as above, that $x$ also is a power of 2.

Now as $x = y + 1$, these results are consistent with each other only if $y = 1$, $x = 2$.

In this last case we have $2^a r + 1 = 2^n - 1^n = 2^r - 1$, since $n = r$. Hence, $2^a(2^{r-a} - r) = 2$, and, therefore, $a = 1$ and $2^{r-a} - r = 1$, that is, $2^{r-1} = r + 1$; so that $r = n = 3$.

(ii) $3^m - 2^m = (2 + 1)^m - 2^m = m2^{m-1} + \binom{m}{2} 2^{m-2} + \cdots + m2 + 1$, and can be of the form $4k + 1$ only if $m$ is even (unless $m = 1$, when $3^m - 2^m = 1$).

But if $m$ is even, $= 2n$, say, $3^m - 2^m = 9^n - 4^n$ and is therefore divisible by $9 - 4$ or 5, the other factor being greater than 1, unless $n = 1$, and is, therefore, prime in this last case only.

(iii) A parallel argument leads to the result that the only primes that are simultaneously of the forms $a^2k + 1$ and $(a + 1)^m - a^m (a > 1)$ are 1 and, possibly, $(a + 1)^a - a^a$. This latter number may or may not be prime as the cases $a = 3, 4$ show.

Also solved by MARY E. CARTER.

**221. (September, 1914.) Proposed by T. E. MASON, Bloomington, Indiana.**

Find a number $x$ such that the sum of the divisors of $x$ is a perfect square. [CARMICHAEL, *Theory of Numbers*, p. 17.]

### SOLUTION BY E. B. ESCOTT, Kansas City, Mo.

(1) Let $x$ be prime. Then $x + 1 = r^2$, or $x = (r + 1)(r - 1)$. This is possible only if $r = 2$; otherwise, $x$ is not prime. Hence, $x = 3$ satisfies the condition.

(2) Let $x = abc \cdots$, where $a, b, c, \cdots$ are different primes. Denoting the sum of the divisors of $x$ by $\Sigma x$,

$$\Sigma x = (a + 1)(b + 1)(c + 1) \cdots.$$

Forming a table of factors of $a + 1$, it is very easy to pick out products of two or more which shall be square, *e. g.*,

| $x$ | $\Sigma x$ | $x$ | $\Sigma x$ |
|---|---|---|---|
| 2.11 | $6^2$ | 7.97 | $28^2$ |
| 2.47 | $12^2$ | · | · |
| 11.47 | $24^2$ | · | · |
| 5.23 | $12^2$ | · | · |
| 5.53 | $18^2$ | 2. 5.7 | $12^2$ |
| 23.53 | $36^2$ | 5. 7.11 | $24^2$ |
| 7.17 | $12^2$ | 5. 7.47 | $48^2$ |
| 7.31 | $16^2$ | 2.13.41 | $42^2$ |
| 7.71 | $24^2$ | 5.67.101 | $204^2$ |
| | | · | · |
| | | · | · |